

SECURE CRYPTOGRAPHIC KEY EXCHANGE  
AND VERIFIABLE DIGITAL SIGNATURE

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BACKGROUND OF THE INVENTIONField of the Invention

The present invention relates generally to cryptography and, more particularly, to exchanging cryptographic keys between two cryptographic units for a single cryptographic session, and to digital signature.

Description of the Prior Art

Two mutually-exclusive classes of cryptographic methods and protocols are well recognized by those familiar with cryptography, symmetric cryptography and public-key cryptography. In symmetric cryptographic protocols, the same key and cryptographic method are used both for encrypting a plaintext message into cyphertext, and for decrypting a cyphertext to recover the plaintext. It is readily apparent that the security of a symmetric cryptographic protocol can never exceed the security of the single key used both for encryption and decryption.

In conventional public-key cryptographic protocols there are two keys, a public key to which anyone can gain access and which is used only for encrypting a plaintext message, and a private key which only the recipient possesses and which is used only for decrypting a cyphertext. For such a public-key cryptographic protocol to be secure it must be unfeasible to determine the

private key by analyzing the public key. While public-key cryptographic systems appear alluring, thus far in practice it has been observed that public-key cryptographic methods are significantly slower than symmetric cryptographic methods. In general, it has been found that public-key cryptographic methods are 1000 times slower than symmetric cryptographic methods. Furthermore, present public key cryptographic methods rely upon difficult but solvable mathematical problems, e.g. factoring large integers or discrete logarithms. Such techniques, while providing some security, can be broken by a cryptanalytic attack that is less exhausting than a brute force attack.

Managing the distribution of cryptographic keys is the most difficult security problem in using cryptography both for symmetric protocols and for public-key protocols. Developing secure cryptographic methods and protocols is not easy, but making sure the keys used with such methods and protocols remain secret is an even more difficult task. "Cryptanalysts often attack both symmetric and public-key cryptosystems through their key management." Schneier, *Applied Cryptography*, Second Edition © 1996 Bruce Schneier ("Schneier") p. 169.

For symmetric cryptographic protocols, there are three well recognized key management problems. First, a key may be compromised which permits an eavesdropper who obtains the key either to read all the cyphertext, or even to broadcast bogus cyphertext.

The only way to alleviate this problem is to change keys frequently. A second problem for symmetric cryptography key management is that it requires a large number of keys if each pair of individuals in a group is to communicate using a different key.

5 Forty-five unique keys are required if a group of 10 individuals are to communicate. Fifty-five unique keys are required for communication among a group of 11 individuals. The final problem for key management in symmetric cryptographic protocols is that, since keys are more valuable than the encrypted messages, the keys  
10 must be exchanged by a secure communication. One approach for securely distributing keys of a symmetric cryptographic protocol is to distribute them using a public-key cryptographic protocol.

Whether used with a symmetric cryptographic protocol or with a public-key cryptographic protocol, an encryption key should not  
15 be used indefinitely. First, the longer a key is used the more likely it will be compromised by theft, luck, extortion, bribery or cryptanalysis. Extended use of a key aids an eavesdropper because that provides more cyphertext encoded with the same key to which  
20 cryptanalytic methods may be applied. Second, in general the longer a key is used the greater the loss if the key is compromised. Accordingly, it is not uncommon to encrypt each individual communication using a separate, session key that is used throughout only one particular communication session.

Schneier at pp. 41-68 provides an overview of protocols for

digital signatures, key exchange, and authentication. Schneier at pp. 513-522 describes in greater detail various key exchange protocols that may be used to establish a session key including:

1. Shamir's Three-Pass protocol which does not use any secret or public keys;
2. a COMSET protocol which uses a public key technique that is equivalent to factoring a large integer; and
3. an Encrypted Key Exchange ("EKE") protocol that may be implemented with various different cryptographic methods such as:
  - a. a Rivest, Shamir and Adleman ("RSA") public-key cryptographic method that is described in United States patent no. 4,405,829;
  - b. an ElGamal public-key cryptographic method; and
  - c. a Diffie-Hellman public-key cryptographic method that is described in United States patent no. 4,200,770.

United States patent nos. 4,405,829 and 4,200,770 together with Schneier are hereby incorporated by reference.

While all of the preceding protocols provide some security for establishing a symmetric cryptographic key, the various protocols require exchanging several, time consuming communications between the parties to establish the key. Moreover, those protocols which require using a public-key cryptographic method also suffer from

the slowness of such methods. Furthermore, the preceding key exchange protocols are no more secure than the cryptographic method which they employed for key exchange, all of which can be broken by cryptanalysis that is less exhausting than a brute force attack.

5        Protocols for key exchange have been developed that are secure against all but a brute force cryptanalytic attack. United States Patent No. 5,583,939 ("the '939 patent") describes an exchange protocol which establish a session key useful for symmetric cryptography:

- 10            1.    employing known and publicly identified mathematical functions; and
2.    applied to exclusively private data, e.g. numbers.

15        In establishing this one-time key, an eavesdropper can learn both some of the numerical values selected by the parties in establishing the key, and also learn some of the numerical values computed using the known and publicly identified mathematical functions. The method disclosed in the '939 patent requires that the four known and publicly identified mathematical functions possess no inverse. That is, the four known and publicly identified functions

20        must possess the property that knowing one of the quantities used in calculating a quantity and the calculated quantity, it is mathematically impossible to compute the other quantity used in performing the calculation. While the method disclosed in the '939 patent is swifter and simpler than previous methods, it requires

initially transmitting at least two quantities between the sender and the receiver, followed by a single quantity between the receiver and the sender.

Another United States Patent No. 5,987,130 ("the '130 patent")  
 5 also describes an exchange protocol which establish a one-time key for use in symmetric cryptography:

1. employing known and publicly identified mathematical functions; and
2. applied to exclusively private data, i.e. numbers.

10 One of the ways in which the method for establishing a one-time key described in the '130 differs from that described in the '939 patent is that an eavesdropper cannot learn any numerical value selected by the parties in establishing the key. That is, the eavesdropper can learn only some of the numerical values computed  
 15 using the known and publicly identified mathematical functions.

For the key exchange protocol described in the '130 patent a first of two cryptographic units "T" and "R" wishing to establish a cryptographic key "K" initially selects a first quantity "A". That same unit then uses a first mathematical function " $\Phi_1$ " and the  
 20 selected quantity "A" to compute a second quantity " $B = \Phi_1(A)$ ". The computed quantity B and the function  $\Phi_1$  must posses the property that knowing the computed quantity B, and the function  $\Phi_1$ , it is mathematically impossible to compute the selected quantity A. That same unit then uses a second mathematical function " $\Phi_2$ " and

the selected quantity "A" to compute a third quantity  $"C" = \Phi_2(A)$ . The first unit T or R which selected the quantity A then transmits the computed quantity B to the other, second unit R or T, while retaining at the first unit T or R the computed quantity C.

5        Upon receiving the quantity B transmitted by the first unit T or R, the second unit R or T first selects a fourth quantity "D." Then using a third mathematical function  $\Phi_3$  together with the selected quantity D, the second unit T or R computes a fifth quantity  $"E" = \Phi_3(D)$ . The computed quantity E and the function  $\Phi_3$  must possess the property that knowing the computed quantity E, and the function  $\Phi_3$ , it is mathematically impossible to compute the selected quantity D. That same unit then using a fourth mathematical function  $\Phi_4$  together with the selected quantity D computes a sixth quantity  $"F" = \Phi_4(D)$ . The second unit R or T which selected  
10 the quantity D then transmits the computed quantity E to the other, first unit T or R, while retaining at the second unit R or T the computed quantity F.

Then the second unit R or T uses a fifth mathematical function  $"\Psi_2"$  together with the calculated quantity F and the received  
20 quantity B to compute the key  $"K" = \Psi_2(F, B) = \Psi_2(\Phi_4\{D\}, \Phi_1\{A\})$ . The first unit T or R upon receiving the quantity E transmitted by the unit R or T then uses a sixth mathematical function  $\Psi_1$  together with the calculated quantity C and the received quantity E to compute the key  $"K" = \Psi_1(C, E) = \Psi_1(\Phi_2\{A\}, \Phi_3\{D\}) = \Psi_2(\Phi_4\{D\}, \Phi_1\{A\})$ .

While the key exchange protocols disclosed both in the '939 and '130 patents permit establishing a session key for symmetric cryptography that an eavesdropper cannot crack except by using a brute force attack, it has not been possible to extend the disclosed techniques for use in digital signatures. The inability to extend the techniques disclosed in the '939 and '130 patents to digital signature appears to arise because the techniques disclosed there avoid using any pre-published, publicly available information in establishing the symmetric cryptographic key. Stated another way, while establishing the cryptographic key each party sends information to the other party on only one occasion, and therefore neither party publishes any information, other than the mathematical functions and the protocol for their use, before establishing the cryptographic key.

#### SUMMARY OF THE INVENTION

An object of the present invention is to provide a cryptographic key exchange protocol which employs pre-established, publicly available information that is provably secure.

Another object of the present invention is to provide a cryptographic key exchange protocol that is faster than conventional protocols.

Another object of the present invention is to provide an encryption key exchange protocol that is secure against all but a

brute force cryptanalytic attack.

Another object of the present invention is to provide an improved, verifiable digital signature.

Another object of the present invention is to provide a  
5 digital signature that is secure against all but a brute force cryptanalytic attack.

10 Briefly, the present invention includes a protocol for cryptographic communication via a communication channel "I" in which a sending cryptographic unit "S" transmits onto the communication channel I an encrypted cyphertext message "M." The sending cryptographic unit "S" obtains the encrypted cyphertext message "M" by supplying both a plaintext message "P" and a cryptographic key "K" to a first cryptographic device. A receiving cryptographic unit "R" receives the cyphertext message M from the communication  
15 channel I, and supplies the cyphertext message M together with the key K to a second cryptographic device. The second cryptographic device decrypts the plaintext message P from cyphertext message M.

In one aspect, the present invention is a method by which the units S and R mutually establish a key K for a cryptographic  
20 session by first exchanging quantities before the sending unit S transmits the cyphertext message M. The method includes the receiving unit R transmitting for storage in a publicly accessible repository a plurality of public quantities. The sending unit S:

1. retrieves the plurality of public quantities from the

publicly accessible repository;

2. using at least some of the plurality of public quantities, computes and transmits to the receiving unit R a plurality of sender's quantities; and

- 5 3. using at least one of the plurality of public quantities, computes the session key K.

The receiving unit R, using at least one of the plurality of sender's quantities received from the sending unit S, computes the session key K.

10 In another aspect, the present invention is a protocol for communication in which a sending unit S transmits onto the communication channel I a message "M" together with a digital signature. However, before transmitting the message M and the digital signature, the sending unit S transmits for storage in the publicly accessible repository a plurality of public quantities.

15 In the method of the present invention, a receiving unit R, that receives the message M and the digital signature, verifies the authenticity of digital signature as follows. The receiving unit R:

- 20 1. retrieves the plurality of public quantities from the publicly accessible repository;
2. using the digital signature and the plurality of public quantities, obtains at least two (2) results by evaluating expressions of at least two (2) different relation-

ships; and

3. compares both pairs of results obtained by evaluating the expressions of the at least two (2) different relationships.

- 5 Finding that the results obtained by evaluating the expressions for at least two (2) different relationships are equal verifies the digital signature.

These and other features, objects and advantages will be understood or apparent to those of ordinary skill in the art from the following detailed description of the preferred embodiment as illustrated in the drawing figure.

#### BRIEF DESCRIPTION OF THE DRAWING

FIG.1 is a block diagram depicting a cryptographic system which may be employed for secure cryptographic key exchange and digital signature via an insecure communication channel.

#### DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENT

FIG. 1 illustrates a cryptographic system which may be employed for cryptographic key exchange that is referred to by the general reference character 10. The cryptographic system 10 includes a sender's cryptographic unit 12a, enclosed within a dashed line, and a receiver's cryptographic unit 12b, also enclosed within a dashed line. One particular unit has been assigned as the

sender's cryptographic unit 12a and another unit has been assigned as the receiver's cryptographic unit 12b only for pedagogical reasons. In principle, either unit could be the sender or the receiver. Each of the cryptographic units 12a and 12b respectively  
5 includes a cryptographic device 14. Each cryptographic device 14 includes a key input port 16, a plaintext port 18, and a cyphertext port 22.

The illustration of FIG. 1 depicts the cyphertext port 22 of the cryptographic device 14 included in the sender's cryptographic unit 12a as being coupled to a first input port 32 of a first transceiver 34a. Consequently, the cyphertext port 22 may supply a cyphertext message "M" to the first transceiver 34a. The first transceiver 34a also includes a first output port 36 from which the first transceiver 34a transmits the cyphertext message M via an  
10 insecure communication channel 38 to a first input port 32 of a second transceiver 34b. The insecure communication channel 38 may include a telephone link, a radio link, a microwave link, a coaxial cable link, a fiber optic link, or any other communication technology that permits transmitting data from a first location to  
15 a second location. Thus, for example, while an electronic or optical communication technology is presently preferred for the insecure communication channel 38, the insecure communication channel 38 might also include a messenger service, or a postal service. For a telephonic insecure communication channel 38, the  
20

transceivers 34a and 34b might each respectively be conventional modems. Upon receipt of the cyphertext message M at the first input port 32 of the second transceiver 34b, the second transceiver 34b transmits the cyphertext message M from a first output port 36 to the cyphertext port 22 of the cryptographic device 14 included in the receiver's cryptographic unit 12b.

Arranged as described above and as illustrated in FIG. 1, the cryptographic units 12a and 12b provide a cryptographic system 10 in which a plaintext message P may be:

1. presented to the plaintext port 18 of the cryptographic device 14 included in the sender's cryptographic unit 12a;
2. encrypted by the cryptographic device 14 into the cyphertext message M;
3. transmitted from the cyphertext port 22 of the cryptographic device 14 via:
  - a. the first transceiver 34a;
  - b. the insecure communication channel 38; and
  - c. the second transceiver 34bto the cyphertext port 22 of the cryptographic device 14 of the receiver's cryptographic unit 12b;
4. decrypted by the cryptographic device 14 back into the plaintext message P; and
5. transmitted from the plaintext port 18 of the crypto-

graphic device 14 included in the receiver's cryptographic unit 12b.

Alternatively, though not illustrated in FIG. 1, the cryptographic system 10 could be arranged so the plaintext message P is transmitted as a cyphertext message M from the cryptographic unit 12b to the cryptographic unit 12a. To effect such a reverse transmission of the plaintext message P, the cyphertext port 22 of the cryptographic device 14 included in the cryptographic unit 12b would be coupled to a second input 42 of the second transceiver 34b rather than to its first output port 36. A second output 44 of the second transceiver 34b would then transmit the cyphertext message M via the insecure communication channel 38 to a second input 42 of the first transceiver 34a. A second output 44 of the first transceiver 34a, rather than its first input port 32, would then be coupled to the cyphertext port 22 of the cryptographic device 14 included in the cryptographic unit 12a. Accordingly, in principle the cryptographic system 10 illustrated in FIG. 1 is capable of being configured for cryptographic transmission of the plaintext message P either from the cryptographic unit 12a to the cryptographic unit 12b as depicted in FIG. 1, or from the cryptographic unit 12b to the cryptographic unit 12a.

The precise cyphertext message M transmitted between the cryptographic units 12a and 12b depends not only upon the plaintext message P, but also upon a particular cryptographic method employed

by the cryptographic device 14 for encryption and/or decryption, and upon a cryptographic key "K" respectively supplied to the key input port 16 of each cryptographic device 14. To supply a cryptographic key K to each cryptographic device 14, both cryptographic units 12a and 12b in accordance with the present invention respectively include a key generator 52 having a key output port 54 from which the key generator 52 transmits the cryptographic key K to the cryptographic device 14.

The cryptographic system 10 depicted in FIG. 1 employs a symmetric cryptographic method for encrypting the plaintext message P, and for decrypting the cyphertext message M. Accordingly, in the illustration of FIG. 1, the cryptographic key  $K^*$  supplied by the key generator 52 to the cryptographic device 14 of the sender's cryptographic unit 12a is identical to the cryptographic key  $K^*$  supplied by the key generator 52 to the cryptographic device 14 of the receiver's cryptographic unit 12b. Described below is the protocol by which the cryptographic units 12a and 12b may mutually establish a cryptographic key  $K^*$  in accordance with the present invention by exchanging messages between the cryptographic units 12a and 12b via the first transceiver 34a, the insecure communication channel 38 and the second transceiver 34b.

Secure Key Exchange

To permit establishing a secure session key to be used during communication between the cryptographic units 12a and 12b, a quantity source 62 included in the receiver's cryptographic unit 12b first generates the following private quantities.

1. a first private, three-element vector  $a = (a_1, a_2, a_3)$
2. a private, large integer 1
3. a second private, three-element vector  $e = (e_1, e_2, e_3)$

The quantity source 62 then transmits the vector  $a$ , the large integer 1, and the vector  $e$  from a quantity output port 64 of the quantity source 62 to a quantity input port 65 of the key generator 52 included in the receiver's cryptographic unit 12b. The quantity source 62 then continues to generate and transmit to the key generator 52 the following single quantity.

4. a first three-element vector  $\alpha = (\alpha_1, \alpha_2, \alpha_3)$

In addition to transmitting  $\alpha$  to the key generator 52, the receiver's cryptographic unit 12b also transmits  $\alpha$  from a publication port 66 of the quantity source 62 for storage in a public repository 67 from which anyone may retrieve it. Numbers in all the quantities listed above are all integers chosen from a finite number set that are preferably obtained using a random number generator. Furthermore, the vector items generated by the quantity source 62, i.e.  $a$ ,  $e$  and  $\alpha$  must be linearly independent.

After the key generator 52 receives the quantities  $a$ , 1,  $e$  and

$\alpha$ , the key generator 52 computes and also transmits to the public repository 67 from a publication port 68 two (2) more quantities listed below.

5. a third three-element vector  $P_1 = 1^a \times (e + \alpha)$

5 6. a fourth three-element vector  $P_2 = 1^e$

When the sender's cryptographic unit 12a wants to establish a secure session cryptographic key K for communication with cryptographic unit 12b, the quantity source 62 of the cryptographic unit 12a generates a private, three-element vector  $r = (r_1, r_2, r_3)$  of random integers chosen from a finite number set. The quantity source 62 transmits the vector  $r$  from the quantity output port 64 to the quantity input port 65 of the key generator 52. After the key generator 52 receives the vector  $r$ , the key generator 52 first retrieves from the public repository 67 through a public-key retrieval-port 69 the three (3) quantities stored there by the receiver's cryptographic unit 12b, i.e.  $\alpha$ ,  $P_1$  and  $P_2$ . Having retrieved those three (3) quantities, the sender's cryptographic unit 12a then computes two vector (2) quantities listed below.

$$V_1 = \alpha \times r$$

20  $V_2 = 1^e \times r$

After computing the two vector quantities  $V_1$  and  $V_2$ , the key generator 52 of the sender's cryptographic unit 12a then transmits them to the receiver's cryptographic unit 12b via an output port 72, the first transceiver 34a, insecure communication channel 38,

second transceiver 34b and an input port 74 of the key generator 52.

After the receiver's cryptographic unit 12b receives the vector quantities  $V_1$  and  $V_2$ , the cryptographic units 12a and 12b then possess all the data needed to independently establish the session cryptographic key K. The receiver's cryptographic unit 12b computes the session cryptographic key K as follows.

$$K = 1^a \cdot (a \times r) 1^e \times r \cdot a$$

The sender's cryptographic unit 12a computes the session cryptographic key K as follows.

$$K = 1^a \times (e + a) \cdot r$$

Because the cryptographic system 10 includes the insecure communication channel 38, an eavesdropper 82, which is not included in the cryptographic system 10 and which is enclosed within a dashed line in FIG. 1, may receive all of the communications between the cryptographic units 12a and 12b. Furthermore, the eavesdropper 82 has access to the public quantities stored in the public repository 67. The eavesdropper 82 includes a cryptographic device 14 which is functionally identical to, and may in principle be the same as, the cryptographic device 14 included both in the cryptographic units 12a and 12b. Therefore, if the eavesdropper 82 were able to determine the cryptographic key K using a key cracker 84 (e.g. by applying an inverse function to the quantities communicated between cryptographic units 12a and 12b during key

exchange and/or to the public quantities stored in the public repository 67) and supply the cryptographic key K to a key input port 16 of the cryptographic device 14, the eavesdropper 82 could decrypt the cyphertext message M to read the plaintext message P.

5 Furthermore, if the eavesdropper 82 possesses the cryptographic key K, the eavesdropper 82 could then also transmit bogus cyphertext message M either to the sender's cryptographic unit 12a, to the receiver's cryptographic unit 12b, or to both.

10 In 1826 Neils Henrik Abel proved that a general equation of fifth or higher order can not be express in terms of radicals. In other words, such an equation can not be solved using purely algebraic means. For the real number system, such an equation can be solved using complex numbers, or a numerical approximation. However, such techniques are inapplicable to discrete, finite  
15 number system used for cryptography.

(C.N., I have created the hypothetical text set forth in the following paragraph from the text in your August 12, 2000, facsimile. Please check its accuracy.)

20 Using all three (3) public quantities, i.e. 4, 5 and 6 above, stored in the public repository 67 by the cryptographic unit 12b and the two vector quantities  $V_1$  and  $V_2$  which the cryptographic unit 12a transmits to the cryptographic unit 12b, solving for a, one private quantities retained by cryptographic unit 12b, requires  
25 solving an equation that includes the vector expression  $a \times (a \times \alpha)$ . Expressing the equation that includes the expression  $a \times (a \times$

α) as a set of simultaneous equations and eliminating any pair of the quantities  $a_1$ ,  $a_2$ , and  $a_3$  yields an 8th order polynomial in the remaining quantity. Since solving analytically for any of the quantities  $a_1$ ,  $a_2$ , and  $a_3$  requires solving an 8th order polynomial, there exists no analytic method for computing the private, three-element vector quantity  $a$  from the public and transmitted quantities.

#### Verifiable Digital Signature

The preceding key exchange protocol may be augmented with additional quantities that allow the cryptographic unit 12b to append verifiable digital signatures to transmitted messages. First, in addition to selecting private, three-element vector  $a$ , the quantity source 62 of the cryptographic unit 12b also selects a large, private integer  $m$  that it provides to the key generator 52. The quantity source 62 then continues to generate and transmit to the key generator 52 a second large integer  $n$ . In addition to transmitting  $n$  to the key generator 52, the receiver's cryptographic unit 12b also transmits  $n$  from the publication port 66 of the quantity source 62 for storage in a public repository 67 from which anyone may retrieve it. Using the integer  $m$ , the key generator 52 then computes and transmits to the public repository 67 three (3) additional public vectors.

7. a fifth three-element vector  $S_1 = a \times (a \times a)$

8. a sixth three-element vector  $S_2 = m^{a \times (a \times (a \times a))}$

9. a seventh three-element vector

$$S_3 = m^{(a \cdot a) (a \times (a \times a))}$$

Having stored the vectors  $S_1$ ,  $S_2$  and  $S_3$  in the public repository 67, the cryptographic unit 12b may then append a digital signature to a message, either the plaintext message P or the cyphertext message M, in the following way. Assuming that the cryptographic unit 12b wants to append a digital signature to the plaintext message P, it first hashes the message P to obtain a three element vector p. After establishing the vector p, the cryptographic unit 12b then appends to the plaintext message P as the digital signature the following three element vector.

$$m^{((a \cdot p)^n) a + a \times p}$$

After retrieving the public quantities that have been stored in the public repository 67, anyone receiving the plaintext message P to which the cryptographic unit 12b has appended the digital signature can verify the signature's authenticity by evaluating and comparing the two following verification expressions.

$$1. \quad m^{(((a \cdot p)^n) a + a \times p) \cdot a \times (a \times a)}$$

$$20 \quad \frac{Q}{T} m^{a \times (a \times (a \times a)) \cdot p}$$

$$2. \quad m^{(((a \cdot p)^n) a + a \times p) \cdot (a \times (a \times a)) \times a}$$

$$\frac{Q}{T} m^{-(a \cdot a) a \times (a \times a) \cdot p}$$

Finding that the quantities obtained by evaluating the two

expressions on both sides of the " $\frac{Q}{P}$ " in verification relationship no. 1 above are identical, and also finding that the quantities obtained by evaluating the two expressions on both sides of the  $\frac{Q}{P}$  in verification relationship no. 2 above are identical, verifies  
 5 the digital signature.

The first expression set forth above prevents a forger from appending a known quantity  $m^{a \times P}$  to the plaintext message P as the signature. The second expression ensures that the cryptographic unit 12b has used the private vector a in computing the digital  
 10 signature.

Considering the public non-linear quantity that includes the term  $a \times (e \times a)$ , solving for the private, three-element vector a requires finding the roots of at least an 8<sup>th</sup> order polynomial. For the reason stated above, there exists no analytic method for  
 15 finding the roots an 8<sup>th</sup> order polynomial. Consequently, cryptanalysts can find the private, three-element vector a only by brute force.

There exist additional expressions which may be used to establish other verification relationships in addition to the two  
 20 set forth above. However, two such verification relationships appear to be sufficient to ensure that the cryptographic unit 12b has appended the digital signature to the message.

Although the present invention has been described in terms of

the presently preferred embodiment, it is to be understood that such disclosure is purely illustrative and is not to be interpreted as limiting. For example, as those skilled in the art will understand, after the cryptographic units 12a and 12b have established the session key K in accordance with the present invention, either of the cryptographic units 12a or 12b may send or may receive cyphertext messages  $M_i$  from the other in any arbitrary order. Analogously, while the digital signature technique may be used with the plaintext message P, it may also be used to authenticate the cyphertext message M. Consequently, without departing from the spirit and scope of the invention, various alterations, modifications, and/or alternative applications of the invention will, no doubt, be suggested to those skilled in the art after having read the preceding disclosure. Accordingly, it is intended that the following claims be interpreted as encompassing all alterations, modifications, or alternative applications as fall within the true spirit and scope of the invention.